**GROUP**

**SEMIGROUP**

**RING**

**FIELD**

**FIBONACCI NUMBER**

THE **FIBONACCI NUMBERS** ARE THE NUMBERS IN THE FOLLOWING [INTEGER SEQUENCE](http://en.wikipedia.org/wiki/Integer_sequence):

BY DEFINITION, THE FIRST TWO FIBONACCI NUMBERS ARE 0 AND 1, AND EACH SUBSEQUENT NUMBER IS THE SUM OF THE PREVIOUS TWO.

**WELL ORDERING PROPERTY**

LET BE A SET OF INTEGERS. AN INTEGER *m* IS CALLED A *LEAST ELEMENT* OF *S* IF *m* IS AN ELEMENT OF *S*, AND FOR EVERY *x* IN *S*, *m≤x*.

**EXAMPLES**

1. 3 IS A LEAST ELEMENT OF THE SET {4,3,5,11}.
2. LET A BE THE SET OF ALL POSITIVE ODD INTEGERS. THEN 1 IS A LEAST ELEMENT OF A.
3. LET U BE THE SET OF ALL ODD INTEGERS. THEN U HAS NO LEAST ELEMENT.

THE PROOF IS LEFT AS AN INFORMAL EXERCISE.

TRICHOTOMY LAW

**ARCHIMEDEAN PROPERTY**

**IF AND ARE ANY POSITIVE INTEGERS, THEN THERE EXISTS A POSITIVE INTEGER N SUCH THAT**

**FIRST PRINCIPLE OF FINITE INDUCTION**

**LET S BE A SET OF POSITIVE INTEGERS WITH THE FOLLOWING PROPERTIES:**

* 1. **THE INTEGER 1 BELONG TO S**
  2. **WHENEVER THE INTEGER K IS IN S, THE NEXT INTEGER K+1 MUST ALSO BE IN S**

**THEN S IS THE SET OF ALL POSITIVE INTEGERS**

**THE PRINCIPLE OF FINITE INDUCTION PROVIDES A BASIS FOR A METHOD OF PROOF CALLED MATHEMATICAL INDUCTION**

**THE BINOMIAL THEOREM**



**WHERE,**

**DIVISIBILITY**

**WHEN AN INTEGER IS DIVIDED BY A SECOND NONZERO INTEGER, THE QUOTIENT MAY OR MAY NOT BE AN INTEGER. FOR EXAMPLE, 24/8 = 3 IS AN INTEGER. WHILE 17/5 = 3.4 IS NOT. THIS LEADS TO THE FOLLOWING THEOREM:**

**DEFINITION**

**IF AND ARE INTEGERS, WE SAY THAT  *DIVIDES b***

***(a |b),*  IF THERE IS AN INTEGER SUCH THAT**

**EXAMPLE: 13 | 182 , -5 | 30, 17 | 289, 7 | 144 (7 does not divide 144)**

**THEOREM (ASSIGNMENT NO 1)**

**For integer a, b and c, the following hold:**



**THEOREM (DIVISION ALGORITHM)**

**LET WITH *THEN, THERE* EXIST UNIQUE INTEGERS AND SUCH THAT**



**THE INTEGERS *q* AND *r* ARE CALLED RESPECTIVELY, THE *QUOTIENT* AND *REMAINDER* IN THE DIVISION OF *a* BY *b*.**

**EXAMPLE**

**FIND THE QUOTIENT AND REMAINDER IN THE DIVISION ALGORITHM WITH DIVISOR 17 AND DIVIDEND 100 AND -100**